MCG Accretion : Limiting Value of η/s

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TCGC−2014

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Chaplygin Gas Accretion Flow : Towards Preprescribed Limit of η over s [η =Shear Viscosity & s = Entropy Density]

by
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Achknowledgement

I want to thank **Prof. Banibrata Mukhopadhyay**, Department of Physics, IISc for the core idea and fruitful discussions.

1. Black Hole Accretion
   - Accretion : What is?
   - Accretion : Where might be found?
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2. Cosmic Acceleration
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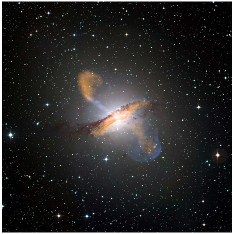

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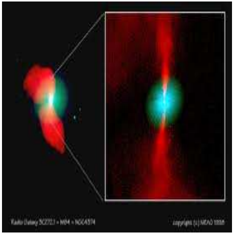

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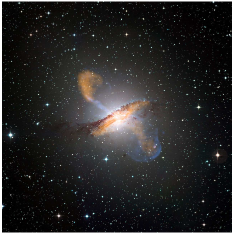

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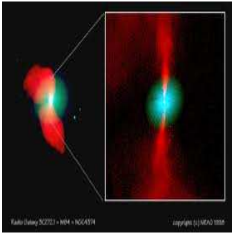
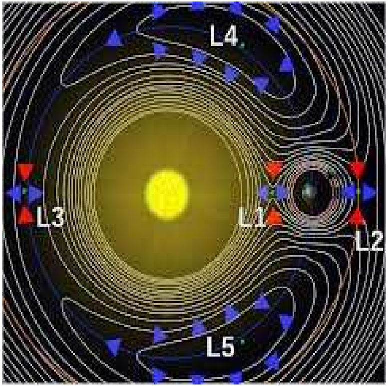


The large luminosity of quasars is believed to be a result of gas being accreted by supermassive black holes. This process can convert about 10% of the mass of an object into energy as compared to around 0.5% for nuclear fusion processes.

Primary Modelling : Accretion Disc

1c



The point L1 : Inner Lagrangian Point, The wind processed by the optical companion when fills the surrounding of it and starts to flow towards the X ray Companion through the L1. This is called Roche Lobe over flow.

Cosmic Acceleration : Modified Chaplygin Gas

- Within the framework of Einstein's gravity, due to this present accelerating phase, it is reasonable to believe that DE is the dominating part (74.5% of the energy content in the observable universe) [1] of the total energy of the universe.

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- A Dark energy candidate MCG has EoS as [2]

$$p = \alpha\rho - \frac{\beta}{\rho^n} \tag{1}$$

It gives the cosmological evolution from an initial radiation era (with $\alpha = \frac{1}{3}$) to (asymptotically) the ΛCDM era (where fluid behaves as cosmological constant).

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MCG around Supermassive Black Holes

- Hoyle first proposed gravitational instability, arising from gravitational coupling with the surrounding matter (tidal interactions), as a possible explanation for galactic rotation. Alternatively, people [3] have proposed that the origin of galaxy rotation could be due to primordial turbulence/vorticity.

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- At present, it is widely believed that the hierarchical clustering of cold dark matter is the origin of structures in the universe. Consequently, the angular momentum of dark matter halos and eventually the rotation of galaxies is thought to be produced by gravitational tidal torque. It has been suggested that the halos obtain their spins through the cumulative acquisition of angular momentum from satellite accretion. DE that has accreted on a galaxy would be similarly torqued by such tidal interactions.

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- Is there any natural site revealing an $\frac{\eta}{s}$ close to above lower bound?

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$$\frac{du}{dx} = \frac{\frac{\lambda^2}{x^3} - F_g(x) + \left(\frac{3}{x} - \frac{1}{F_g}\frac{dF_g}{dx}\right)\frac{c_s^4}{\left\{(1-n)c_s^2+\alpha(n+1)\right\}}}{u - \frac{2c_s^4}{u\left\{(1-n)c_s^2+\alpha(n+1)\right\}}} = \frac{N}{D}.$$

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- $u\frac{d\lambda}{dx} = \frac{2\sqrt{F}\alpha_s}{x^{\frac{1}{2}}c_s}\left[\frac{u^2-c_s^2+(n+1)\alpha_s}{n} - x(c_s^2+u^2)\frac{1}{n+1}\frac{c_s}{c_s^2-\alpha_s}\frac{dc_s}{dx} + xu\frac{du}{dx}\right]$

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- We will fix the critical point distance, i.e., we will provide the energy of the accreting object. $N = D = 0$ relation with particular r_c will give us the radial velocity and the critical point sound speed. We will fix λ_c manually.

The Angular Momentum Gradient

- $u\frac{d\lambda}{dx} = \frac{2\sqrt{F}\alpha_s}{x^{\frac{1}{2}}c_s}\left[\frac{u^2-c_s^2+(n+1)\alpha_s}{n} - x(c_s^2+u^2)\frac{1}{n+1}\frac{c_s}{c_s^2-\alpha_s}\frac{dc_s}{dx} + xu\frac{du}{dx}\right]$
- Relativistic flow must transit to supersonic phase from subsonic via sonic point/ critical point.At the critical point the denominator of the velocity gradient becomes zero. To make a continuous flow we should have numerator zero as well and to use L'Hospital's rule to obtain the sonic point velocity gradient.
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Solutions : Spherical Accretion

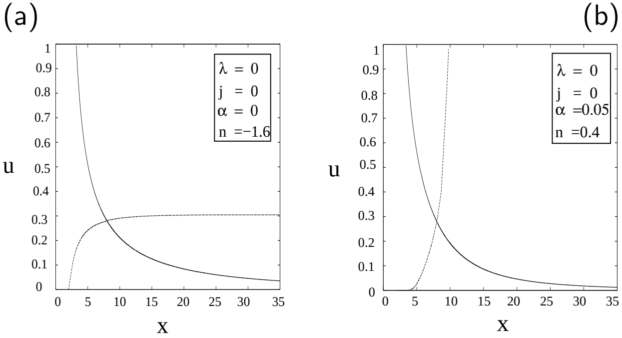

Fig. 1(a) & 1(b) represent the variation of accretion and wind speeds as functions of radial coordinate for $\lambda = 0$, $j = 0$. The solid line represents the accretion speed whereas the rest is for wind speed. It should be marked that the absolute value of the velocities are been plotted here.

Solutions : Disc Accretion, Non Rotating BH

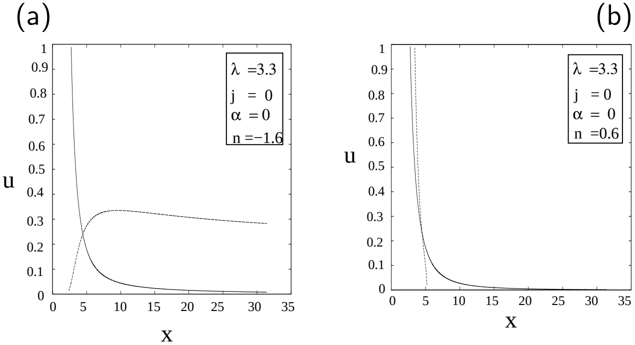

Fig. 2(a) & 2(b) represent the variation of accretion and wind speeds in disk flows as functions of radial coordinate for $j = 0$. The solid line represents the accretion speed whereas the rest is for wind speed. It should be marked that the absolute value of the velocities are been plotted here.

Solutions : Disc Accretion, Rotating BH

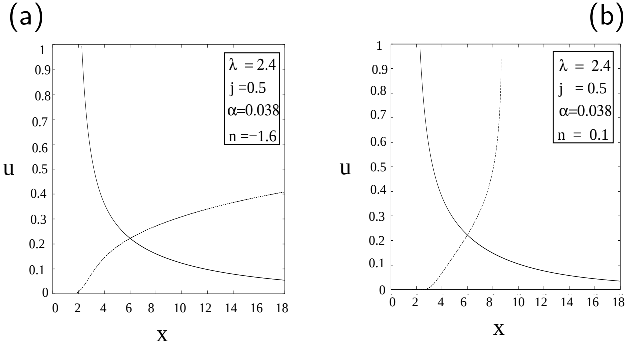

Fig. 3(a) & 3(b) represent the variation of accretion and wind speeds in disk flows as functions of radial coordinate for $j = 0.5$. The solid line represents the accretion speed whereas the rest is for wind speed. It should be marked that the abosolute value of the velocities are been plotted here.

Solutions : $\frac{\eta}{s}$

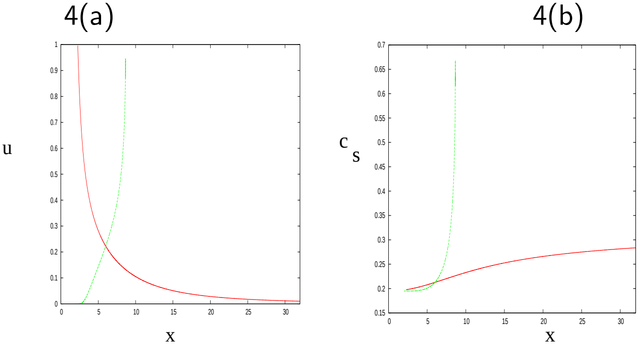

Fig. 4(a) & 4(b) represent the variation of accretion and wind radial velocity (red) and sound speeds (green) in disk flows as functions of radial coordinate for $j = 0.5$, $\lambda_c = 2.4$, $\alpha_{SS} = 0.01$, $T = 7 \times 10^{11}\ K(ion\ temp)$ as we have assumed optically thin and geometrically thick flow.

Solutions : $\frac{\eta}{s}$

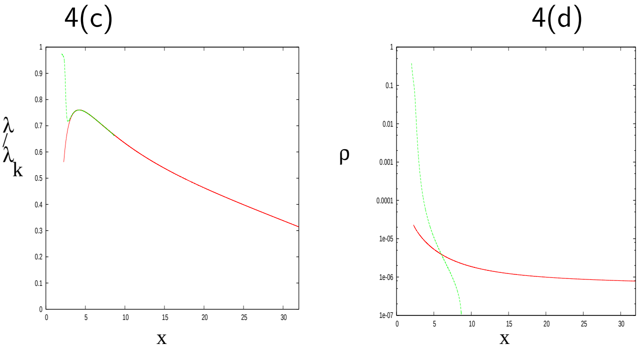

Fig. 4(c) & 4(d) represent the variation of accretion and wind $\frac{\lambda}{\lambda_k}$ and density (10^{-18}gm/cc unit) in disk flows as functions of radial coordinate. All other parameters are been given the previous case.

Solutions : $\frac{\eta}{s}$

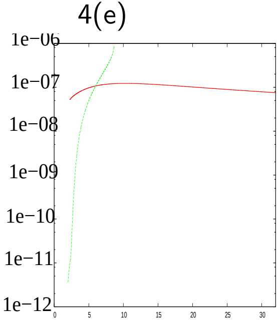

Fig. 4(e) represents the variation of accretion and wind $\frac{\eta}{s}$ for disk flows as functions of radial coordinate. $\frac{\eta}{s} \sim 4 \times 10^{-12}$ at the wind branch very near to the BH. The lower limit required 6×10^{-13}.

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THANK YOU!