EFFECT OF COSMIC BACKREACTION ON AN ACCELERATING UNIVERSE

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INTRODUCTION

- ✓ The cause of the current acceleration is attributed to "Dark Energy".
- ✓ The true nature of Dark Energy is unknown.
- ✓ Observations also tell us that the Universe is inhomogeneous up to scales of super-clusters of galaxies.
- ✓ Backreaction from inhomogeneities could modify the evolution of the Universe.
- ✓ Backreaction could lead to an accelerated expansion during the present epoch.

THE BACKREACTION FRAMEWORK

1) T. Buchert, Gen. Rel. Grav., 32, 105 (2000); T. Buchert, Gen. Rel. Grav. 33, 1381. (2001).

2) A. Wiegand, T. Buchert, Phys. Rev. D 82, 023523 (2010).

For a spatial domain $\mathcal{D}$, the scale factor is defined as

$$a_{\mathcal{D}}(t) = \left(\frac{|\mathcal{D}|_g}{|\mathcal{D}_i|_g} \right)^{1/3}$$

It encodes the average stretch of all directions of the domain.

The Einstein equations turn out to be

$$\begin{aligned} 3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= -4\pi G \langle \rho \rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} + \Lambda \\ 3H_{\mathcal{D}}^2 &= 8\pi G \langle \rho \rangle_{\mathcal{D}} - \frac{1}{2}\langle \mathcal{R} \rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{Q}_{\mathcal{D}} + \Lambda \\ 0 &= \partial_t \langle \rho \rangle_{\mathcal{D}} + 3H_{\mathcal{D}} \langle \rho \rangle_{\mathcal{D}} \end{aligned}$$

Average of the scalar quantities on the domain $\mathcal{D}$ is

$$\langle f \rangle_{\mathcal{D}}(t) = \frac{\int_{\mathcal{D}} f(t, X^1, X^2, X^3)\, d\mu_g}{\int_{\mathcal{D}} d\mu_g}$$

ρ = local matter density

$\mathcal{R}$ = Ricci-scalar

$H_{\mathcal{D}}$ = domain dependent Hubble rate

The kinematical backreaction $\mathcal{Q}_{\mathcal{D}}$ is defined as

$$\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left(\langle \theta^2 \rangle_{\mathcal{D}} - \langle \theta \rangle_{\mathcal{D}}^2\right) - 2\sigma_{\mathcal{D}}^2$$

where θ is the local expansion rate, and $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$ is the squared rate of shear.

The "global" domain $\mathcal{D}$ is assumed to be separated into sub-regions, which themselves consist of elementary space entities.

The acceleration equation becomes

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \sum_{\ell} \lambda_{\ell} \frac{\ddot{a}_{\ell}(t)}{a_{\ell}(t)} + \sum_{\ell \neq m} \lambda_{\ell} \lambda_{m} \left(H_{\ell} - H_{m} \right)^2$$

where λ_{ℓ} is the volume fraction of the sub-domain

We only work with two sub-regions:

$\mathcal{M}$ – those parts that have initial overdensity (called “Wall”)
$\mathcal{E}$ – those parts that have initial underdensity (called “Void”)

This allows us to write the acceleration equation as

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \lambda_{\mathcal{M}} \frac{\ddot{a}_{\mathcal{M}}}{a_{\mathcal{M}}} + \lambda_{\mathcal{E}} \frac{\ddot{a}_{\mathcal{E}}}{a_{\mathcal{E}}} + 2\lambda_{\mathcal{M}}\lambda_{\mathcal{E}}\left(H_{\mathcal{M}} - H_{\mathcal{E}}\right)^2$$

EVOLUTION WITHIN THE FRAMEWORK

1) N. Bose, A. S. Majumdar, Gen. Rel. Grav. 45:1971-1987 (2013).

The scale factors of the two sub-regions $\mathcal{M}$ and $\mathcal{E}$ are assumed to be

$$a_{\mathcal{E}} = c_{\mathcal{E}} t^{\alpha} \; ; \; a_{\mathcal{M}} = c_{\mathcal{M}} t^{\beta}$$

Volume fraction of the sub-domain $\mathcal{M}$ is, $\lambda_{\mathcal{M}} = \dfrac{|\mathcal{M}|_g}{|\mathcal{D}|_g} = \dfrac{a_{\mathcal{M}}^3 |\mathcal{M}_i|_g}{a_{\mathcal{D}}^3 |\mathcal{D}_i|_g}$

The acceleration equation for $\mathcal{D}$ can now be written as

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \frac{g_{\mathcal{M}_h}^3 t^{3\beta}}{a_{\mathcal{D}}^3} \frac{\beta(\beta-1)}{t^2} + \left(1 - \frac{g_{\mathcal{M}_h}^3 t^{3\beta}}{a_{\mathcal{D}}^3}\right) \frac{\alpha(\alpha-1)}{t^2} + 2 \frac{g_{\mathcal{M}_h}^3 t^{3\beta}}{a_{\mathcal{D}}^3} \left(1 - \frac{g_{\mathcal{M}_h}^3 t^{3\beta}}{a_{\mathcal{D}}^3}\right) \left(\frac{\beta}{t} - \frac{\alpha}{t}\right)^2$$

EFFECT OF EVENT HORIZON

1) N. Bose, A. S. Majumdar, MNRAS: Letters 418: L45--L48 (2011).
2) N. Bose, A. S. Majumdar, Gen. Rel. Grav. 45:1971-1987 (2013).

The “global” domain $\mathcal{D}$ is large enough to be considered homogeneous. This allows us to write

$$a_{\mathcal{D}} \approx c_F a_F \;\; ; \;\; H_{\mathcal{D}} \approx H_F$$

In the same spirit we can write the event horizon for the global domain as

$$r_h = a_{\mathcal{D}} \int_t^{\infty} \frac{dt'}{a_{\mathcal{D}}\left(t'\right)}$$

Only those regions of $\mathcal{D}$ that are within the event horizon are accessible to us. Hence apparent volume fraction of $\mathcal{M}$ is

$$\lambda_{\mathcal{M}_h} = \frac{a_{\mathcal{M}}^3 \, | \mathcal{M}_i |_g}{\frac{4}{3}\pi r_h^3} = \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3}$$

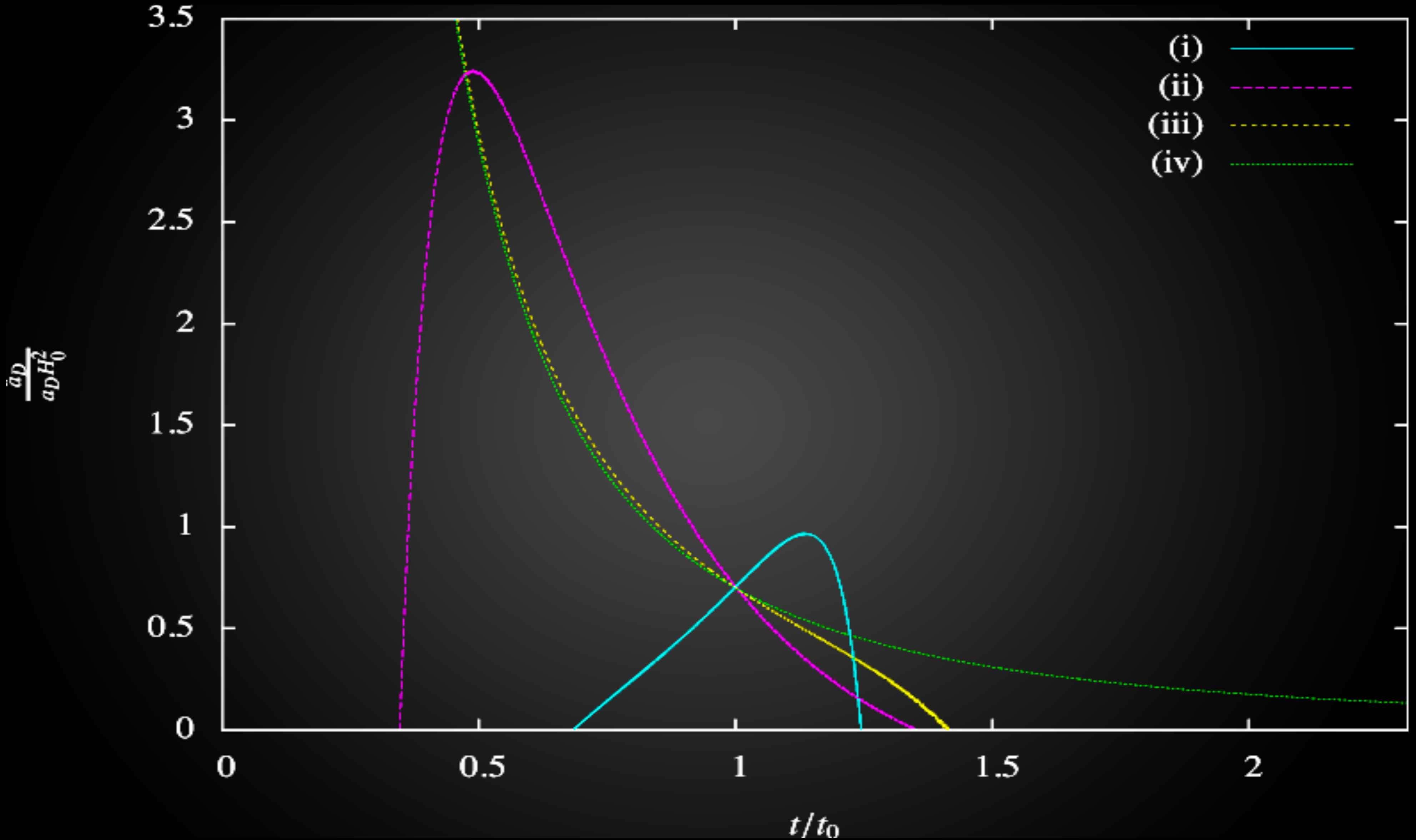
And that for $\mathcal{E}$ is $\lambda_{\mathcal{E}_h} = 1 - \lambda_{\mathcal{M}_h}$

Convert Event Horizon equation

$$r_h = a_{\mathcal{D}} \int_t^{\infty} \frac{dt'}{a_{\mathcal{D}}\left(t'\right)} \quad \rightarrow \quad \dot{r}_h = \frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} r_h - 1$$

Numerically solve along with the global acceleration equation

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3} \frac{\beta(\beta-1)}{t^2} + \left(1 - \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3}\right) \frac{\alpha(\alpha-1)}{t^2}$$
$$+ 2 \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3} \left(1 - \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3}\right) \left(\frac{\beta}{t} - \frac{\alpha}{t}\right)^2$$



(i) $\alpha = 0.995$, $\beta = 0.5$, (ii) $\alpha = 0.984$, $\beta = 0.5$,
(iii) $\alpha = 1.02$, $\beta = 0.66$, (iv) $\alpha = 1.02$, $\beta = 0.66$

BACKREACTION USING MULTIPLE DOMAINS

1) N. Bose, A. S. Majumdar, arXiv:1306.2877 [astro- ph.CO].

The "global" domain $\mathcal{D}$ partitioned into equal numbers of overdense and underdense domains

$$\mathcal{D} = \left(\cup_j \mathcal{M}^j\right) \cup \left(\cup_j \mathcal{E}^j\right)$$

The scale factors of the sub-regions $\mathcal{E}^j$ and $\mathcal{M}^j$ are assumed to be

$$a_{\mathcal{E}_j} = c_{\mathcal{E}_j} t^{\alpha_j} \; ; \; a_{\mathcal{M}_j} = c_{\mathcal{M}_j} t^{\beta_j}$$

Volume fraction of the sub-domain $\mathcal{M}^j$ is, $\lambda_{\mathcal{M}_j} = \dfrac{|\mathcal{M}^j|_g}{|\mathcal{D}|_g} = \dfrac{a^3_{\mathcal{M}_j} |\mathcal{M}^j_i|_g}{a^3_{\mathcal{D}} |\mathcal{D}_i|_g}$

And similarly for the $\mathcal{E}^j$ sub-domains

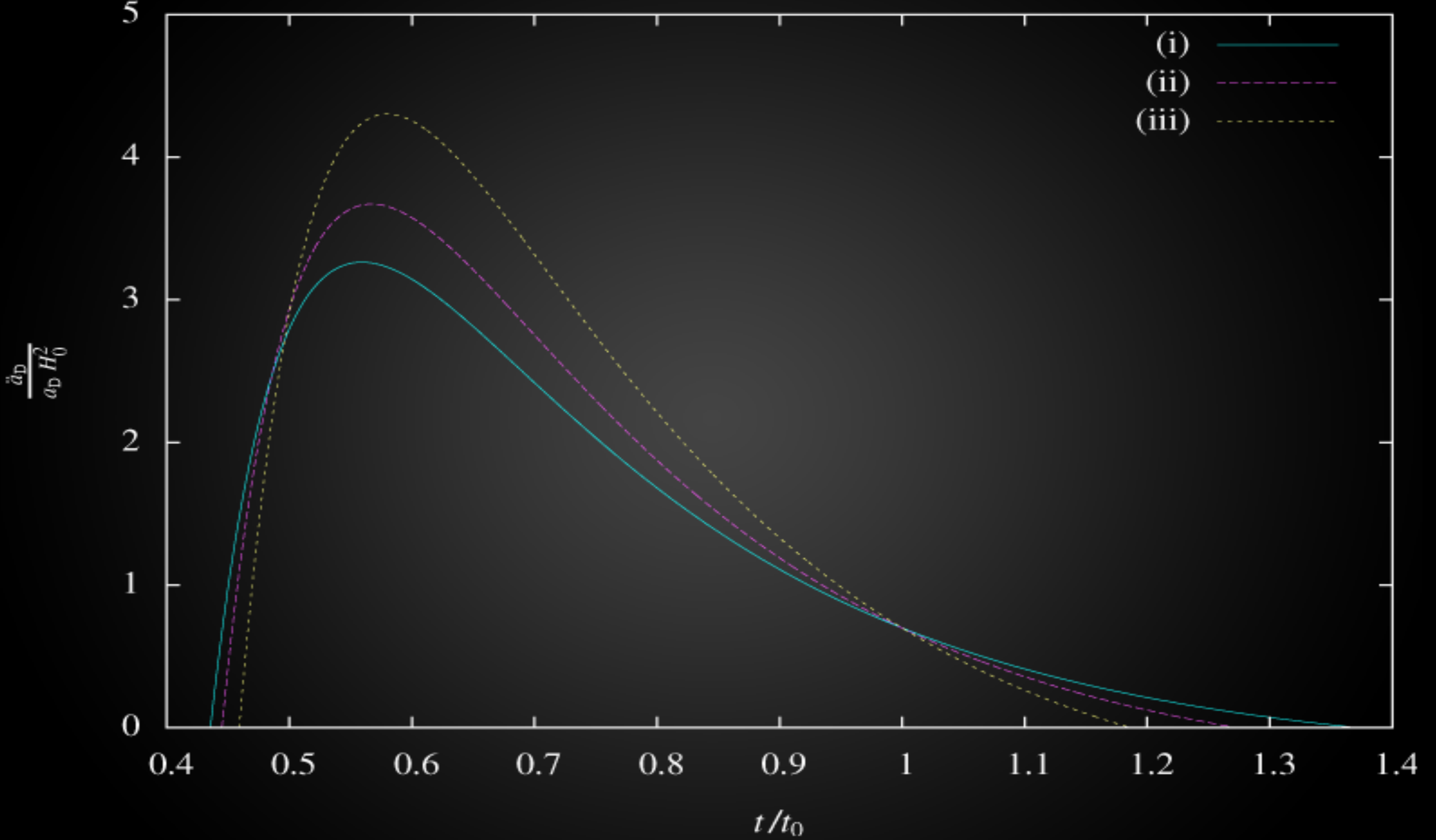
The global acceleration therefore becomes

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \sum_j \frac{g^3_{\mathcal{M}_j} t^{3\beta_j}}{a^3_{\mathcal{D}}} \frac{\beta_j(\beta_j - 1)}{t^2} + \sum_j \frac{g^3_{\mathcal{E}_j} t^{3\alpha_j}}{a^3_{\mathcal{D}}} \frac{\alpha_j(\alpha_j - 1)}{t^2}$$

$$+ \sum_{j \neq k} \frac{g^3_{\mathcal{M}_j} t^{3\beta_j}}{a^3_{\mathcal{D}}} \frac{g^3_{\mathcal{M}_k} t^{3\beta_k}}{a^3_{\mathcal{D}}} \left(\frac{\beta_j}{t} - \frac{\beta_k}{t} \right)^2$$

$$+ \sum_{j \neq k} \frac{g^3_{\mathcal{E}_j} t^{3\alpha_j}}{a^3_{\mathcal{D}}} \frac{g^3_{\mathcal{E}_k} t^{3\alpha_k}}{a^3_{\mathcal{D}}} \left(\frac{\alpha_j}{t} - \frac{\alpha_k}{t} \right)^2$$

$$+ 2 \sum_{j,k} \frac{g^3_{\mathcal{M}_j} t^{3\beta_j}}{a^3_{\mathcal{D}}} \frac{g^3_{\mathcal{E}_k} t^{3\alpha_k}}{a^3_{\mathcal{D}}} \left(\frac{\beta_j}{t} - \frac{\alpha_k}{t} \right)^2$$



α_j is in the range 0.990 - 0.999, and β_j in the range 0.58 - 0.60. For curve (i), (ii) and (iii) we consider 100, 400 and 500 overdense and underdense subdomains each, respectively.

CONCLUSION

- ✓ We have explored the effect of backreaction on the evolution of the accelerating Universe.
- ✓ The presence of the cosmic event horizon causes the acceleration to slow down significantly with time.
- ✓ If the Universe is accelerating due to a different mechanism then backreaction will eventually cause it slow down.

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2) “Effect of cosmic backreaction on the future evolution of an accelerating universe” – Nilok Bose, A. S. Majumdar
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3) “Study of cosmic backreaction on the future evolution of an accelerating universe using multiple domains” – Nilok Bose, A. S. Majumdar
arXiv:1306.2877 [astro- ph.CO], in preparation