BH Thermodynamics with Cosmic Acceleration

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TCGC−2014

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Effects of Cosmic Acceleration on Black Hole Thermodynamics

by
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$$dE = \frac{k}{8\pi} dA + \Omega dJ + \phi dQ \quad .$$

Where E is the energy, k is the surface gravity, A is the horizon area, Ω is the angular velocity, J is the angular momentum, ϕ is the electrostatic potential and Q is the electric charge.

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Quintessence

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Fig.1a

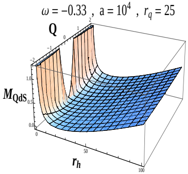


Fig.1b

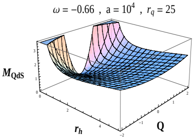
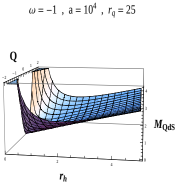


Fig.1c



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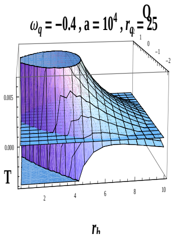

Fig.2b

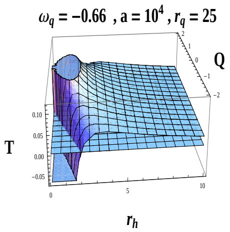

Fig.2c

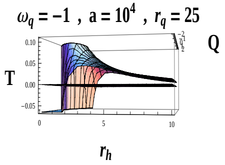

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Fig.3a

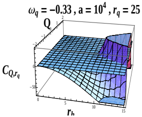

Fig.3b

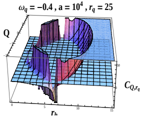

Fig.3c

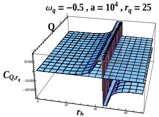

Fig.3d

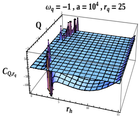

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Fig.4a

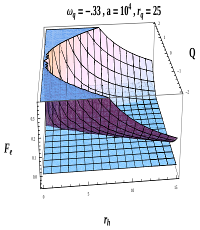


Fig.4b

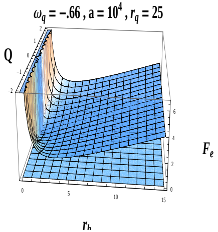
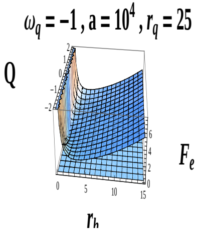


Fig.4c



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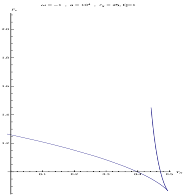


Fig.5b

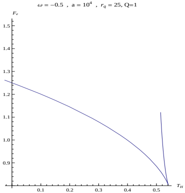


Fig.5c

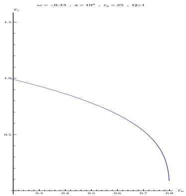
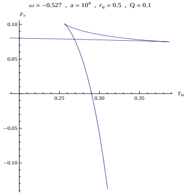


Fig.5d



Figures 5a-d represent the variation of free energy with respect to T_H for the different parameters set.

Fig 5a-5d show the variation of free energy with Hawking temperature. In all the cases except $\omega_q = -0.33$ we can find at least one cuspidal type double point where $\frac{\partial F_e}{\partial T_H}$ has single value but that occurs twice. This thing signifies a Hawking Page phase transition.

Conclusions

- So the obligatory sketch out of charged BH thermodynamics with the central engine amplified into a quintessence field is stable small BH $\rightarrow$ Second order phase transition $\rightarrow$ Unstable small/intermediate mass BH $\rightarrow$ First order phase transition $\rightarrow$ Stable intermediate mass BH $\rightarrow$ Second order phase transition $\rightarrow$ unstable super massive BHs.

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THANKS