Violation of Kubo-Martin-Schwinger condition along a Rindler trajectory in polymer quatization

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September 19, 2015

References

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2. "Absence of Unruh effect in polymer quantization".
   G. M. Hossain and G. Sardar, arXiv:1411.1935 [gr-qc].

Out line of talk

- Introduction
- Rindler-Space time
- KMS Condition
- Two-point function in Minkowski spacetime
- Fock quantization
- Polymer quantization
- Two point function along Rindler trajectory in Fock quantization
- Two point function along Rindler trajectory in polymer quantization
- Discussion

Introduction

- What is Unruh Effect?
  With respect to a *uniformly accelerating* observer, Fock vacuum state appears as a *thermal* state rather than a zero-particle state

- Theoretical approaches
  - Bogolubov transformation
  - Model detectors
  - KMS periodicity

Rindler Space-Time

- Rindler metric

  $ds^2 = e^{2a\xi}(-d\tau^2 + d\xi^2) + dy^2 + dz^2$

  where a is magnitude of 4-acceleration

- Minkowski metric

  $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

- Relation Between Rindler and Minkowski Co-ordinates

  $x = \frac{e^{a\xi}}{a} Sinh(a\tau)$

  $t = \frac{e^{a\xi}}{a} Cosh(a\tau)$

KMS Condition

- Gibbs ensemble average
  $\langle \hat{O} \rangle_\beta = Z^{-1} Tr \left[e^{-\beta \hat{H}} \hat{O} \right]$
  where $\beta = 1/k_B T$ and $Z = Tr \left[e^{-\beta \hat{H}} \right]$
- For $\hat{O} = \hat{\phi}(\tau, \vec{\xi})\hat{\phi}(\tau', \vec{\xi'})$
  $\langle \hat{\phi}(\tau, \vec{\xi})\hat{\phi}(\tau', \vec{\xi'}) \rangle_\beta = Z^{-1} Tr \left[e^{-\beta \hat{H}} \hat{\phi}(\tau, \vec{\xi})\hat{\phi}(\tau', \vec{\xi'}) \right]$
- KMS condition
  $\langle \hat{\phi}(\tau, \vec{\xi})\hat{\phi}(\tau', \vec{\xi'}) \rangle_\beta = \langle \hat{\phi}(\tau', \vec{\xi'})\hat{\phi}(\tau + i\beta, \vec{\xi}) \rangle_\beta$

Two-point function in Minkowski spacetime

- Two-point function
  $G(x, x') \equiv \langle 0|\hat{\Phi}(x)\hat{\Phi}(x')|0\rangle = \langle 0|\hat{\Phi}(t, \mathbf{x})\hat{\Phi}(t', \mathbf{x}')|0\rangle$
- Using definition of Fourier modes
  $G(x, x') = \frac{1}{V}\sum_{\mathrm{k}} D_{\mathrm{k}}(t, t')\ e^{i\mathrm{k}\cdot(\mathbf{x}-\mathbf{x}')}$
  where $D_{\mathrm{k}}(t, t') = \langle 0_{\mathrm{k}}|e^{i\hat{\mathcal{H}}_{\mathrm{k}}t}\hat{\phi}_{\mathrm{k}}e^{-i\hat{\mathcal{H}}_{\mathrm{k}}t}e^{i\hat{\mathcal{H}}_{\mathrm{k}}t'}\hat{\phi}_{\mathrm{k}}e^{-i\hat{\mathcal{H}}_{\mathrm{k}}t'}|0_{\mathrm{k}}\rangle$
- $\hat{\phi}_{\mathrm{k}}|0_{\mathrm{k}}\rangle = \sum_n c_n|n_{\mathrm{k}}\rangle$
- $D_{\mathrm{k}}(t - t') \equiv D_{\mathrm{k}}(t, t') = \sum_n |c_n|^2 e^{-i\Delta E_n(t-t')}$
  where $\Delta E_n \equiv E_n^{(\mathrm{k})} - E_0^{(\mathrm{k})}$

Two point function Fock quantization

- Energy spectra
  $E_{\mathrm{k}}^{n} = \left(n + \frac{1}{2}\right)|\mathrm{k}| \Rightarrow \Delta E_n = (E_n^{(\mathrm{k})} - E_0^{(\mathrm{k})}) = n|\mathrm{k}|$

- $c_n = \langle n_{\mathrm{k}}|\hat{\phi}_{\mathrm{k}}|0_{\mathrm{k}}\rangle$
  $c_n = \frac{\delta_{1,n}}{\sqrt{2|\mathrm{k}|}}$

- Two-point function
  $G(x,x') = \frac{(1-i\epsilon)^{-1}}{4\pi^2\Delta x^2}$

where $\Delta x^2 = -\Delta t^2 + |\Delta \mathbf{x}|^2$

Energy spectra(asymptotic)in polymer quantization

- In low-energy regime $(g \ll 1; g = |k| l_\star)$

$$\frac{\Delta E_{4n+3}}{|\mathrm{k}|} = (2n+1) - \frac{(4n+3)^2 - 1}{16} g + \mathcal{O}\left(g^2\right)$$

for $n \geq 0$

$$c_3 = \frac{i}{\sqrt{2|\mathrm{k}|}} \left[1 + \mathcal{O}\left(g\right)\right] \ , \ \frac{c_{4n+3}}{c_3} = \mathcal{O}\left(g^n\right)$$

for $n > 0$

- In high-energy regime $(g \gg 1)$

$$\frac{\Delta E_{4n+3}}{|\mathrm{k}|} = 2(n+1)^2 g + \mathcal{O}\left(\frac{1}{g^3}\right) \text{ for } n \geq 0$$

$$c_3 = i\sqrt{\frac{g}{2|\mathrm{k}|}} \left[\frac{1}{4g^2} + \mathcal{O}\left(\frac{1}{g^6}\right)\right], \frac{c_{4n+3}}{c_3} = \mathcal{O}\left(\frac{1}{g^{2n}}\right)$$

for $n > 0$

Two point function in polymer quantization

- Domain of perturbation
  $l_\star \Delta t \ll (\Delta t \pm |\Delta \mathbf{x}|)^2$

- Polymer corrected two point function

$$G(x,x') \simeq \frac{(1-i\epsilon)^{-1}}{4\pi^2 \Delta x^2}\left[1 + \frac{2i\ \delta^{poly}\ l_\star \Delta t}{(1-i\epsilon)\Delta x^2}\right]$$

where $\delta^{poly} = 2\ \delta_{c3} + \delta_{E3}\left[1 + 4(\Delta t^2/\Delta x^2)\right]$

KMS condition in Rindler frame

- Thermal two-point function wrt Rindler obserber
  $\mathcal{G}(\tau, \tau') \equiv \langle \hat{\phi}(\tau, \vec{\xi}_0) \hat{\phi}(\tau', \vec{\xi}_0) \rangle_\beta$
- KMS condition
  $\mathcal{G}(\tau, \tau') = \mathcal{G}(\tau', \tau + i\beta)$
- $\mathcal{G}(\tau) \equiv \mathcal{G}(0, \tau) \implies \mathcal{G}(-\tau) = \mathcal{G}(\tau + i\beta)$

Fockspace two point function along the rindler trajectory

- Trajectory of the detector
  $x_d(\tau) = (\sinh a\tau/a, \cosh a\tau/a, 0, 0)$
- Two point function along the rindler trajectory
  $G(\tau) \equiv G\left(x_d(\tau), x_d(0)\right) = \frac{a^2(1-i\epsilon)^{-1}}{8\pi^2(1-\cosh a\tau)}$
- Satisfy KMS condition
  $G(-\tau) = G(\tau + i\beta)$ with $\beta = 2\pi/a$

Fock vacuum appears like a *thermal reservoir* of temperature $T = a/2\pi k_B$ which is precisely equal to Unruh temperature.

Polymer corrected two point function along the rindler trajectory

- Polymer corrected two point function along the rindler trajectory

$$G^{poly}(\tau) = \frac{a^2(1-i\epsilon)^{-1}}{8\pi^2(1-\cosh a\tau)}\left[1+\Delta G_\star^{(1)}+\mathcal{O}(l_\star^2)\right]$$

where $\Delta G_\star^{(1)} = \frac{il_\star a\sinh a\tau[2\delta_{c3}-\delta_{E3}(1+2\cosh a\tau)]}{(1-i\epsilon)(1-\cosh a\tau)}$

- Does not satisfy KMS condition
$G^{poly}(-\tau) \neq G^{poly}(\tau+i\beta)$

Summary & Discussions

- In Fock quantization, two point function in Minkowski vacuum along a Rindler trajectory follows KMS periodicity with $\beta = 2\pi/a$
  So,the corresponding reservoir temperature is $T = a/2\pi k_B$

- In polymer quantization, two pont function in Minkowski vacuum along Rindler trajectory does not follows KMS condition.
  Energy spectram of the the fourier modes is the primar cause of KMS violation.
  Can be think a non periodic function as a periodic function with infinite periodicity. So, the corresponding temperature $T = 0$.

Thank You