The role of gravity in Physics beyond standard model of elementary particles

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Principle of Naturalness

1. A quantity in nature can be naturally very small only if the underlying theory becomes more symmetric as that quantity tends to zero – t'hooft
2. The symmetry protects the quantity against a possible large quantum corrections upto the cut-off scale of the theory
3. Cutoff scale is the scale where some new Physics enters without which the description is incomplete
4. For example : The fermion masses in SM are protected to a small value – due to underlying chiral symmtery – not true for Higgs scalar

Naturalness problem in standard model

1. standard model keeps gravity away because gravity is extremely weak at the electroweak scale
2. The standard model, despite it's tremendous success, is therefore bound to fail near a cut-off scale where gravity becomes strong called 'Planck scale'
3. Large hierarchy between the electroweak and Planck scale – leads to large mass correction to the scalar Higgs which is not protected by any symmetry
4. To keep it within 1 Tev, unnatural fine tuning is necessary in every order of perturbation theory

What is Planck scale ?

Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

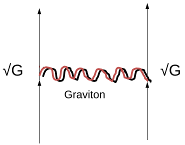
Consider a small fluctuation over flat space metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G} h_{\mu\nu}$$

Gravitational coupling to a scalar field

$$g_{\mu\nu}\partial^{\mu}\Phi\partial^{\nu}\Phi = \partial_{\mu}\Phi\partial^{\mu}\Phi \ + \ \sqrt{G} h_{\mu\nu}\partial^{\mu}\Phi\partial^{\nu}\Phi$$

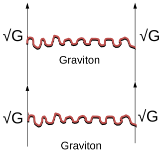
Graviton exchange amplitude



The amplitude $\sim (E^2 G)$ in natural units

Gravity is relevant at energy scale $E \sim \frac{1}{\sqrt{G}} = 10^{19} Gev = M_{Planck}$
Thus Planck scale is a natural cut-off for SM

Graviton loop,



Amplitude $\sim \frac{\Lambda^4}{M^4}$ —- UV divergent and non-renormalizable – Problem of quantum gravity

$$m_H^2 = m_0^2 + 3\frac{\Lambda^2}{8\pi^2 v^2}(m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2)$$

This implies

$$\delta m_H^2 \sim \Lambda^2$$

where Λ is the cutoff scale say Planck scale

To keep m_H within Tev, one needs extreme fine tuning $\sim 10^{-32}$

UNNATURAL

Challenge for standard model?

Supersymmetry

1. Bring in bosonic (fermionic) partner for every fermionic (bosonic) SM particles
2. Assume that the masses of the superpartners are same as their SM counterpart and demand bose-fermi exchange symmetry
3. This cancels the quadratic divergences in the Higgs mass correction – no fine tuning is necessary – naturalness restored
4. No superpartner is observed so far, implying SUSY is a broken symmetry
5. SUSY breaking at Tev scale indicates generation of vacuum energy $\sim$ Tev^4 – far far above the observed value

Cosmological constant and fine tuning problem

Recall Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

Lorentz invariance demands that the vacuum energy-momentum tensor is of the

form $T_{\mu\nu} = -\rho g_{\mu\nu}$ implying $\lambda_{eff} = \lambda + 8\pi G \rho$

Observational bound on effective vacuum energy density $\rho_v = \frac{\lambda_{eff}}{8\pi G} \sim 10^{-47} Gev^4$

Vacuum energy in standard model – Naturalness problem again !

Consider the scalar field potential in SM,

$$V = V_0 - \mu^2 \phi^+ \phi + g(\phi^+ \phi)^2$$

$$\rho = V_{min} = V_0 - \frac{\mu^4}{4g} = V_0 - 10^6 Gev^4 \qquad (1)$$

This implies V_0 must be tuned to 53 place of decimal to get the desired value –

UNNATURAL !!

Supergravity

1. Demand local SUSY invariance
2. Gravitino appears as the partner of graviton. The SM fields and their partners are described by superfields Φ
3. Theory is described by three functions Kahler potential $K(\Phi, \Phi^+)$, Superpotential $W(\Phi)$ and a gauge kinetic function
4. Breaking of local SUSY (say in some hidden sector) is mediated to break SUSY in observable sector – in terms of gravitino mass
5. $V = e^G(G^i(G^{-1})^j_i G_j - 3)$ where $G = K + W$
6. $V_{min} = 0$ i.e vacuum energy vanishes by extreme fine tuning again !!

Gravity in higher dimension and Naturalness

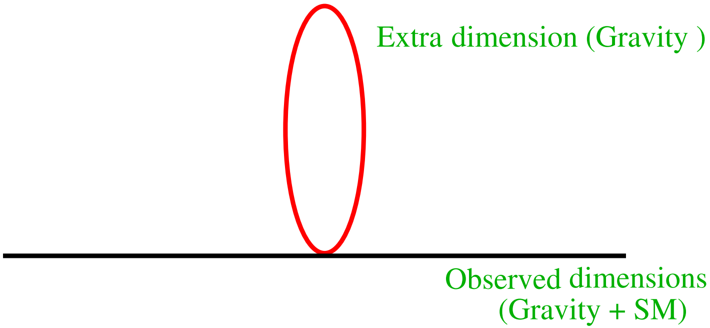
We have seen that 4-dimensional gravity can not have any role in BSM Physics at low energy apart from setting a cut-off for the theory

However the scenario changes drastically in presence of extra spatial dimensions

We now explore these possibilities

Large extra dimensions - ADD model

ADD Scenario:



Einstein action in d dimensions:
$S = \frac{1}{16\pi G_d} \int d^d x \sqrt{-g_d}\; R_d$

Assume:

$$ds_d{}^2 = ds^2_4{}_{Observed} - dy_I dy^I{}_{Unobserved}$$

Then

$$S = \frac{V_{d-4}}{16\pi G_d} \int d^4 x \sqrt{-g_4}\; R_4 \quad = \frac{1}{16\pi G_4} \int d^4 x \sqrt{-g_4}\; R_4$$

$G_4 = \frac{G_d}{V_{d-4}}$

Four dimensional (observed) Planck scale
$M_{Pl(4)} = 10^{19} GeV$

$[M_{Pl(d)}^{d-2} = (\frac{1}{L})^{d-4} M_{Pl(4)}^2]$

(i) $d = 6$, $L = 100\ \mu m \quad \Rightarrow \quad M_{Pl(6)} = 1\ TeV$

(ii) $d = 10$, $L = 1\ Fermi \quad \Rightarrow \quad M_{Pl(10)} = 1\ TeV$

Instead of the question why m_W is small compared M_P now we have the question why V_n is so large? – Hierarchy in a new guise !
Stabilizing mechanism of this large volume has been proposed in the context of string theory by using fluxes – Fine tuning again ?

Warped Geometry – Randall-Sundrum Model

The Einstein action in 5 dimensional ADS_5 space

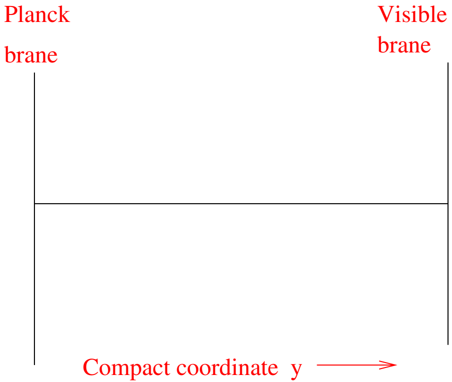
$$S = \frac{1}{16 G_5} \int d^5x \sqrt{-g_5} \, [\mathcal{R} - \Lambda]$$

Compactify the extra coordinate $y = r\phi$ on S_1/Z_2 orbifold

Identify ϕ to $-\phi$ i.e lower semi-circle to upper semi circle

Place two 3-branes at the two orbifold fixed points $\phi = 0, \pi$

r is the radius of S_1



The Z_2 orbifolded coordinate $y = r\phi$ with $0 \leq \phi \leq \pi$ and r is the radius of the S_1

Action

$$S = S_{Gravity} + S_{vis} + S_{hid}$$

$$S_{Gravity} = \int d^4x \; r \; d\phi \sqrt{-G} \; [2M^3 R - \underbrace{\Lambda}_{5-dim}]$$

$$S_{vis} = \int d^4x \sqrt{-g_{vis}} \; [L_{vis} - V_{vis}]$$

$$S_{hid} = \int d^4x \sqrt{-g_{hid}} \; [L_{hid} - V_{hid}]$$

Metric ansatz:

$$ds^2 = e^{-A(\phi)}\ \eta_{\mu\nu} dx^\mu dx^\nu + r^2 d\phi^2$$

Warp factor and the brane tensions are found by solving the 5 dimensional

Einstein's equation with orbifolded boundary conditions

$$A = 2kr\phi$$

$$V_{hid} = -V_{vis} = 24M^3 k$$

and

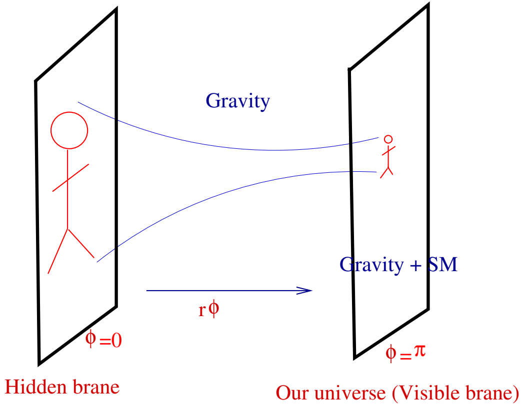
$$k^2 = \frac{-\Lambda}{24M^3}$$

Warping

$$\left(\frac{m_H}{m_0}\right)^2 = e^{-2A}|_{\phi=\pi} = e^{-2kr\pi} \sim (10^{-16})^2$$

$\Rightarrow kr = \frac{16}{\pi}\ln(10) = 11.6279 \quad \leftarrow$ RS value with $k \sim M_P$ and $r \sim l_P$

So hierarchy problem is resolved not by introducing any new scale but by diluting the scale through a warped geometry



Origin of warped model – Effective Einstein's equation – an 'induced gravity' description

1. Consider a system of two 3-branes placed at the orbifold fixed points and embedded in a bulk
2. Bulk is a five dimensional AdS spacetime containing the bulk cosmological constant Λ_5 only
3. The most general metric is taken through radion field ϕ which is a function of both spacetime co-ordinates x^μ and extra dimensional co-ordinate y

Metric Ansatz

$$ds^2 = q_{\mu\nu}(y,x)dx^\mu dx^\nu + e^{2\phi(y,x)}dy^2$$

The proper distance between the two branes within the fixed interval $y = 0$ to $y = r\pi$ is given by:

$$d_0(x) = \int_0^{r\pi} dy e^{\phi(y,x)}$$

1. Putting a brane in a bulk space-time induces an effective Einstein's equation on the brane due to the bulk curvature

2. It may be derived using Gauss-Codacci equation with appropriate junction condition where brane-bulk curvature ratio as perturbing parameter

The Einstein's equations in first order on visible brane:

$$
\begin{aligned}
{}^{(4)}G^{\mu}_{\nu} &= \frac{\kappa^2}{l}\frac{1}{\Phi}\,T^{\mu}_{2\ \nu} + \frac{\kappa^2}{l}\frac{(1+\Phi)^2}{\Phi}\,T^{\mu}_{1\ \nu} \\
&+ \frac{1}{\Phi}\left(D^{\mu}D_{\nu}\Phi - \delta^{\mu}_{\nu}D^2\Phi\right) \\
&+ \frac{\omega(\Phi)}{\Phi^2}\left(D^{\mu}\Phi D_{\nu}\Phi - \frac{1}{2}\delta^{\mu}_{\nu}(D\Phi)^2\right)
\end{aligned}
$$

$$
Radion\ \Phi = e^{2d_0/l} - 1, \qquad \omega(\Phi) = -\frac{3}{2}\frac{\Phi}{1+\Phi}
$$

Φ is a function of the brane co-ordinates x

1. It may be shown that the effective four dimensional cosmological constant $\Lambda_4 = (V_{vis} + \sqrt{-24\Lambda M^3})$
2. Tune $V_{hid} = -V_{vis} = \sqrt{-24\Lambda M^3}$ to have a flat brane with metric $\eta_{\mu\nu}$ —- Constant radion scenario
3. Thus in RS model, the combined effect of bulk cosmological constant and the brane tensions on the 3-branes are tuned exactly to counterbalance one another to produce a vanishing brane cosmological constant such that the visible brane is flat
4. But to produce a vacuum energy $\sim 10^{-47}$, we need to fine tune two terms whose values are $\sim 10^{76}$ – fine tuning at 123-rd place of decimal !!
5. The fate is similar when warped geometry models are construcetd in a string background – 'Throat geometry' with fluxes tuned unnaturally to produce the desired de-Sitter vacuum.
6. Can a brane-bulk non-alignment occur naturally to produce a small but non-vanishing vacuum energy?

Warped braneworld with non-zero cosmological constant

The metric :

$$ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

The action is :

$$S = \int d^5x \sqrt{-G}(M^3 \mathcal{R} - \Lambda_5) + \int d^4x \sqrt{-g_i}\, \mathcal{V}_i$$

Solving The bulk Einstein's equations away from the 3-branes we have,:

$$^{(4)}G_{\mu\nu} = -\Omega g_{\mu\nu}$$

This is the effective four dimensional Einstein's equation with Ω as the induced cosmological constant on the brane

Solution for the warp factor

$$6A'^2 = -\frac{\Lambda_5}{2M^3} + 2\Omega e^{2A}$$

$$3A'' = \Omega e^{2A} \qquad (2)$$

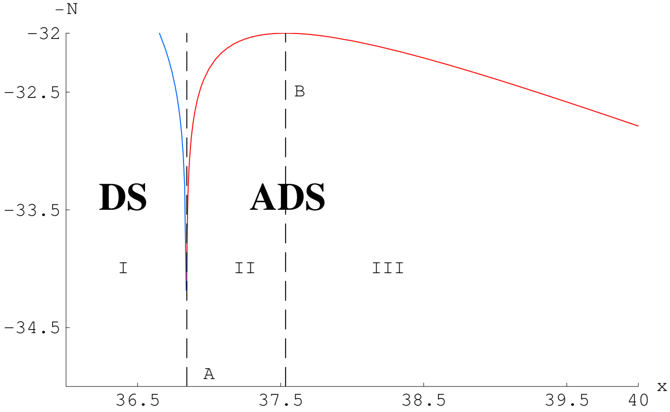
Assume ω^2 **is positive – de Sitter**

$$e^{-A} = \omega \sinh\left(\ln \frac{c_2}{\omega} - k|y|\right)$$

where $\omega^2 = \Omega/3k^2$ with $c_2 = 1 + \sqrt{1+\omega^2}$

For $\omega \to 0$ we retrieve RS solution

We have the entire parameter space of $\omega^2 = 10^{-N}$ and $kr\pi = x$ which produces the desired warping of 10^{-16}



The brane tensions on both the branes are:

$$\mathcal{V}_{vis} = -12M^3\tilde{k}\left[\frac{c_2^2+\omega_{vis}^2}{c_2^2-\omega_{vis}^2}\right], \mathcal{V}_{pl} = 12M^3\tilde{k}\left[\frac{c_2^2+\omega_{pl}^2}{c_2^2-\omega_{pl}^2}\right]$$

The contribution to non-zero value of induced cosmological constant ω_{vis}^2 on the

visible brane comes from the fine tuned imbalance between first order correction to the extrinsic curvature and projected Weyl tensor and the brane tension

Unlike RS model with a negative tension visible brane (intrinsically unstable), here we can have two positive tension branes

Modulus stabilization

1. In RS model the modulus can be stabilised by Goldberger-Wise mechanism by using a scalar field in the bulk and tuning the boundary values such that
2. $rk \sim \frac{k^2}{m^2} log \frac{\Phi_P}{\Phi_T} \sim 12$
3. But no back-reaction of the bulk field is taken and the inclusion of the bulk scalar action is quite ad-hoc
4. Can we include scalar back-reaction? Can we find a geometric origin of this ?

Signature of RS model –Massive towers

1. Bulk graviton KK modes of mass at Tev range
2. Massless graviton mode couples to standard model fields at the brane as $\sim 1/M_P$
3. Massive graviton modes couple $\sim e^{kr\pi}/M_P \sim Tev^{-1}$
   (Though the number of KK modes are smaller in number than ADD model, but has enhanced coupling)
4. Detectable signatures at LHC?

The conflict with LHC results

1. In the search for extra dimension through dilepton events in 8-TeV proton-proton collision, the ATLAS detector at LHC has set stringent lower bound on the mass of the Randall-Sundrum (RS) lightest graviton Kaluza-Klein (KK) mode ~ 2.5 Tev
2. we take $k < M$ with $M \sim M_{Pl}$. so that k, which measures the bulk curvature must be smaller than the Planck scale ensuring the validity of the classical solutions for the bulk metric given by RS model.

Consider the metric expansion,

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

we can find the KK mass tower of graviton from this expression

$$m_n = x_n k e^{-k r_c \pi}$$

Also recall

$m_H = m_0 e^{-k r_c \pi}$

Using $\epsilon = k/M_{Pl}$,

$$m_1 = x_1 \epsilon \frac{m_H}{m_0} M_{Pl}$$

$M_{Pl}^2 = \frac{M^3}{k}$

Take $m_0 = \alpha M$

M is the 5-dimensional Planck scale and α is any constant parameter $\alpha = 1$

implies that the cut-off scale is the quantum gravity scale $M \sim M_{Pl}$, while $\alpha < 1$ indicates the appearence of new physics below Planck scale.

$$m_1 = x_1 \epsilon^{2/3} \frac{m_H}{\alpha}$$

If we consider α to be 1 i.e the cut-off scale is the 5-dimensional Planck scale M then values of m_1 for different values of ϵ varying from $0.01 - 0.1$ are

$\epsilon = \frac{k}{M_{Pl}}$	$m_1 = x_1 \epsilon^{2/3} m_H$(GeV)
0.01	22.39
0.03	46.59
0.05	65.49
0.07	81.96
0.09	96.91
0.1	103.96

Table: Theoretical values of first KK mass mode of graviton From RS model when $\alpha = 1$, $x_1 = 3.83$ and $m_H = 126.0\mathrm{GeV}$

The Experimental lower bound for the mass of the first KK mode of graviton for different values of ϵ as reported by the ATLAS Collaboration are shown in table:

$\epsilon = k/\overline{M}_{Pl}$	m_1(TeV)
0.01	1.01
0.03	1.48
0.05	1.88
0.07	2.04
0.09	2.17
0.1	2.22

Table: The mass table from the results of ATLAS

The tables clearly indicate that for the entire range of $0.01 < \epsilon < 1$, the theoretical prediction for the mass of the first KK mode of graviton is much below the lower bound set by the ATLAS data.

For a possible resolution to this problem we calculate the threshold values of the parameter α from the expression, $\alpha = x_1 \epsilon^{2/3} m_H / m_1$ and use the values of lower bound of m_1 for different ϵ as reported by ATLAS data.

These values are shown in table:

$\epsilon = k/M_{Pl}$	m_1 from ATLAS(TeV)	values of α
0.01	1.01	2.2×10^{-2}
0.03	1.48	3.1×10^{-2}
0.05	1.88	3.4×10^{-2}
0.07	2.04	4.0×10^{-2}
0.09	2.17	4.4×10^{-2}
0.1	2.22	4.6×10^{-2}

1. The scale m_0 is nearly two order lower then the Planck scale
2. This will further go down if the future experiments raise the lower bound of the mass of lightest KK graviton mode even more.
3. A possible resolution is to assume that r_c is few order larger than l_{Pl} such that $m_0 \sim r_C^{-1}$ But that brings back intermediate scale again.
4. More number of warped dimensions with larger number of moduli ?

Higher curvature Gauss-Bonnet gravity

The RS model considers a 5-dimensional Einstein gravity action with a cosmological term

Space-time dimension higher than four in general admits of suitable combinations of higher order curvature terms when added to Einstein gravity still lead to second order field equations which in turn ensures the model to be free of any ghost field

In 5-dimensions, only a particular combination of the curvature terms, called the Gauss-Bonnet (GB) term gives rise to the most general ghost-free theory for gravity

Such a term appears as the correction at leading order in the inverse string tension, to the gravity action in string theory

A term of this type in the action turns out to be a trivial surface term in 4-dimensions

Such an addition can in principle modify phenomenological and cosmological signatures significantly

The characteristic parameter of Einstein-Gauss-Bonnet (EGB) theory is the coefficient of the higher derivative terms, denoted here as α

In the context of warped phenomenology, the GB correction modifies the conventional RS model by giving rise to an α-dependent warp factor

The overall framework is described by the following action

$$
\begin{aligned}
S_5 &= S_{EH} + S_{GB} + S_{Brane} + S_{Bulk} \\
S_{EH} &= \frac{M^3}{2}\int d^5x\sqrt{-g_{(5)}}R_{(5)} \\
S_{GB} &= \frac{\alpha M}{2}\int d^5x\sqrt{-g_{(5)}}[R^{ABCD}_{(5)}R^{(5)}_{ABCD} - 4R^{AB}_{(5)}R^{(5)}_{AB} + R^2_{(5)}] \\
S_{Brane} &= \int d^5x\sum_{i=1}^{2}\sqrt{-g^{(i)}_{(5)}}[\mathcal{L}^{field}_i - T_i]\delta(y-y_i) \\
S_{Bulk} &= \int d^5x\sqrt{-g_{(5)}}[\mathcal{L}^{field}_{Bulk} - 2\Lambda]
\end{aligned}
$$

In the above action, i is the Brane index, i=1(Hidden brane), 2(Visible brane) and $\mathcal{L}^{field}_i$ is the Lagrangian for the fields on the ith brane with the brane tension T_i. Similarly, $\mathcal{L}^{Bulk}$ is the Lagrangian for the fields present in the bulk. All SM fields are on the visible brane, as they are open string modes in string-inspired scenarios.

The 5-dimensional metric assumes the form

$$ds^2 = e^{-2A(y)}\eta_{\alpha\beta}dx^\alpha dx^\beta + r_c^2 dy^2$$

Integrating out the coordinate y from the 5-dimensional action, we arrive at

$$\bar{M_{Pl}}^2 \simeq \frac{M^3}{k_\alpha}$$

In terms of k_{RS} we have

$$k_\alpha = \sqrt{\frac{3M^2}{16\alpha}[1 - \sqrt{1 - \frac{32\alpha k_{RS}^2}{3M^2}}]}$$

The reality of k_α demands an upper bound on α, given by

$$\alpha \leq \frac{3}{32}\frac{M^2}{k_{RS}}^2$$

With the GB correction, the expression for graviton kk-mode mass is modified to

$$m_n = k_\alpha x_n e^{-k_\alpha r_c \pi}$$

where m_n is the mass of the n^{th} mode and x_n is the nth root of $J_1(x)$. In our

modified model with GB correction the coupling of graviton KK-modes with SM fields becomes

$$\Lambda_\pi^{-1} = \frac{e^{k_\alpha r_c \pi}}{\bar{M}_{Pl}}$$

Despite the change in the value of the parameter 'k' due to the GB coupling α, the magnitude of the exponent is kept in the range 11.4-11.7, to achieve the desired hierarchy

This keeps the graviton KK mode coupling with the brane fields similar to that in RS model, with k_α replacing k_{RS}

If we consider the first KK mode of RS graviton then its mass is given by

$$m_G = k_\alpha e^{-k_\alpha r_c \pi} x_1$$

where $x_1 = 3.83$. From the above relation, increase in m_G implies increase in k_α for a fixed value of the warp factor. A suitable value of α can raise the first KK mode mass beyond the lower bound set by ATLAS without disturbing the condition $k_{RS}/M < 1$

F(R) gravity as higher curvature correction

Now we turn our attention to another class of quantum gravity corrections namely $F(R)$ model
We start from the following action for $F(R)$ gravity on the bulk

$$S = \int d^5x\sqrt{-G}\left(M^3 F(R) - \Lambda\right) + \int d^4x\sqrt{-g_i}V_i$$

where Λ is the bulk cosmological constant, R is the five dimensional Ricci scalar and V_i is the brane tension for ith brane.
The warped metric ansatz is:

$$ds^2 = e^{-2A(y)}g_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2$$

The Einstein equation in $F(R)$ gravity with constant scalar curvature are:,

$$\left\{ {}^4R_{\mu\nu} + \frac{e^{-2A}}{r_c^2}\left(A'' - 4A'^2\right) g_{\mu\nu} \right\} F'(R) - \frac{1}{2} g_{\mu\nu} e^{-2A} F(R) = -\frac{\Lambda}{2M^3} e^{-2A} g_{\mu\nu}$$

$$\frac{4}{r_c^2}\left(A'' - A'^2\right) F'(R) - \frac{1}{2} F(R) = -\frac{\Lambda}{2M^3}$$

and the five dimensional scalar curvature has the following expression, ${}^5R = e^{2A}({}^4R) + \frac{1}{r_c^2}\left(8A'' - 20A'^2\right)$.

From these equations spacetime part and the extra dimension part separate as,

$$
\begin{aligned}
{}^4G_{\mu\nu} &= \Omega g_{\mu\nu} \\
3A'' &= \Omega r_c^2 e^{2A}
\end{aligned}
$$

Introducing $F(R) = R + f(R)$ (with rescaling $y \to r_c y$),

$$
\begin{aligned}
(A')^2 \Big(6 &- 4f'\Big) \\
&= -\frac{\Lambda}{2M^3} + \frac{f}{2} + 2\Omega e^{2A}\left(1 - \frac{2}{3} f'\right)
\end{aligned}
$$

From the above equation we could obtain the following solution for the variable A as,

$$
\begin{aligned}
e^{-A} &= \omega \cosh\left(ln\frac{\omega}{c_1} + k_F y \right) \\
k_F^2 &= -\frac{1}{6}\left(\frac{\frac{\Lambda}{2M^3} - \frac{f}{2}}{1 - \frac{2}{3}f'} \right)
\end{aligned}
$$

The bulk cosmological constant Λ is negative for anti-deSitter bulk
$f'(R)$ imply derivative of the function $f(R)$ with respect to R

The respective brane tensions are being given by,

$$
\begin{aligned}
V_{vis} &= 12M^3 k_F \left[\frac{\frac{\omega^2}{c_1^2} e^{2kr_c\pi} - 1}{\frac{\omega^2}{c_1^2} e^{2kr_c\pi} + 1} \right] \\
V_{hid} &= 12M^3 k_F \left[\frac{1 - \frac{\omega^2}{c_1^2}}{1 + \frac{\omega^2}{c_1^2}} \right]
\end{aligned}
$$

As usual The graviton KK modes has the following expansion

$$h_{\alpha\beta}(x,\phi) = \sum_{n=0}^{\infty} h_{\alpha\beta}^{(n)}(x) \frac{\chi^{(n)}(\phi)}{\sqrt{r_c}}$$

The masses of the graviton KK excitations

$$m_n = k_F x_n e^{-k_F r_c \pi}$$

The usual form of the interaction Lagrangian is,

$$L = \frac{1}{M^{3/2}} T^{\alpha\beta}(x) h_{\alpha\beta}(x, \phi = \pi)$$

Expanding the graviton field into the KK states and using the proper normalization for $\chi_n(\phi)$ we arrive at,

$$L = \frac{1}{M_{Pl}} T^{\alpha\beta} h^{(0)}_{\alpha\beta} - \frac{1}{\Lambda_\pi} \sum_{n=1}^{\infty} T^{\alpha\beta} h^{(n)}_{\alpha\beta}$$

Thus in the context of $F(R)$ gravity as well the zero mode separates out from the sum and couples with inverse 4-dimensional scale, M_{Pl}.

All the massive KK states are also suppressed by the scale $\Lambda_\pi = e^{k_F r_c \pi} M_{Pl}$, which is of order the inverse weak scale which is same as in RS model.

Hence the same lower bound of first KKmass of 3 TeV also applies in this context.

Now we would like to know what is the criteria for k_F^2 to be greater than k_{RS}^2 so that the graviton KK mass can exceed the lower bound set by ATLAS.

Using $f(R) = \beta R^n$ we obtain the bound,

$$R > -\frac{2n \mid \Lambda \mid}{3M^3}$$

The criteria $k_F/M < 1$ (i.e. bulk curvature is less than 5D Planck scale to ensure that the classical solution can be trusted) leads to an inequality given by

$$\frac{\mid \Lambda \mid}{12M^5} + \frac{\beta R^{n-1}}{2M^2}\left\{R + \frac{4}{3}nM^2\right\} < 1$$

An ADS bulk implies, $\beta > 0$ is the valid choice for odd powers of R, while $\beta < 0$ is a valid choice for even powers of R.

Hence the form of $F(R)$ should be
$F(R) = R - \beta R^2$ or $R + \beta R^3$ etc.

Thus the graviton KK mode bounds by recent LHC experiments can not be explained in standard RS scenario, it can be explained quiet well in $F(R)$ gravity framework AND we find constraints on

Bulk curvature and Nature of the $F(R)$ gravity model

We have seen that the coupling of the graviton KK- modes with SM fields remain same as RS -scenario

The graviton KK mode mass expression

$$m_n = k_F x_n e^{-k_F r_c \pi}$$

Also

$$k_F^2 = -\frac{1}{6}\left(\frac{\frac{\Lambda}{2M^3} - \frac{f}{2}}{1 - \frac{2}{3}f'}\right)$$

Suitable choice of $f(R)$ can increase the value of k_F to raise the 1-st graviton KK-mode mass beyond 3 Tev

A geometric modulus stabilization

We start by postulating the five-dimensional pure gravity action, in the Jordan frame, to be

$$S_{EH} = \int d^4xdy\sqrt{\tilde{g}}(2M^3 f(\tilde{R}) - 2\lambda M^5)$$
$$- \int d^4xdy\sqrt{\tilde{g}}[\lambda_v\delta(y-\pi) + \lambda_h\delta(y)],$$

Concentrating on the bulk action, it can be rewritten as

$$S_{blk} = \int d^4xdy\sqrt{\tilde{g}}(2M^3\tilde{R}\,F - U - 2\lambda M^5), \tag{3}$$

where,

$$U = 2M^3[\tilde{R}F - f(\tilde{R})]F \equiv f'(\tilde{R}).$$

The non-minimal coupling above can be rotated away by a conformal transformation,

$$\tilde{g}_{ab} \to g_{ab} = \exp(2\,\omega(x^\mu, y))\,\tilde{g}_{ab}$$

The Ricci scalars in the two frames are related through

$$\tilde{R} = e^{2\,\omega}\left[R + 8\,\Box\omega - 12 g^{ab}\partial_a\omega\partial_b\omega\right] \ ,$$

with $\Box$ representing the Laplacian operator appropriate for the Einstein frame (defined in terms of g_{ab}).
Choose a specific form of $\omega(x^\mu, y)$,

$$\omega = \frac{1}{3}\,\ln F \equiv \frac{\gamma\,\phi}{5}\ , \qquad \gamma \equiv \frac{5}{4\,\sqrt{3}\,M^{3/2}}\ , \tag{4}$$

This leads to

$$S = \int d^4x dy \sqrt{g}[2M^3 R - \frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi - V(\phi)] \\ - \int d^4x dy \sqrt{g} e^{-\gamma\phi} [\lambda_v \delta(y - l) + \lambda_h \delta(y)]$$

where

$$V(\phi) = [U(\phi) + 2\lambda M^5] \exp(-\gamma\phi) \ . \tag{5}$$

1. We have traded the complex form of $f(\tilde{R})$ for the usual Einstein-Hilbert action, supplemented by a scalar field that encapsulates the extra degree of freedom encoded in the higher powers of derivatives in $f(\tilde{R})$
2. As long as the potential $V(\phi)$ is bounded from below, the system would be free from instabilities
3. The exact form of $V(\phi)$ would, of course depends on the form of $f(\tilde{R})$
4. Consider the case where $V(\phi)$ has a minimum at $\phi = \phi_{\rm min}$. Given sufficient time, one would expect that ϕ would settle at ϕ_{min} with $V(\phi_{min})$ acting as the effective cosmological constant (*i.e*, it would assume the role of Λ
5. To the leading order, only small deviations about ϕ_{min} should be considered

Defining $y \equiv r_c y$, we consider the metric

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

The Einstein's equations reduce to

$$\begin{aligned} 6\sigma'^2 &= \frac{1}{4M^3}[\frac{1}{2}\phi'^2 - V] \\ 3\sigma'' &= \frac{1}{4M^3}\left[\phi'^2 + e^{-\gamma\phi}(\lambda_h\delta(y) + \lambda_v\delta(y-l))\right] \end{aligned} \tag{6}$$

The scalar field satisfies,

$$\phi'' - 4\sigma'\phi' - \frac{dV}{d\phi} + \gamma e^{-\gamma\phi}[\lambda_h\delta(y) + \lambda_v\delta(y-l)] = 0 \qquad (7)$$

Expanding around $\phi = \phi_a \sim \phi_{\min}$,

$$\frac{V}{M^5} = V_0 + (\frac{V_1}{M^{7/2}})\xi + (\frac{V_2}{M^2})\xi^2, \qquad (8)$$

where $\xi(y) = M^{-3/2}(\phi - \phi_a)$ and V_i are constants.

We may now substitute this form of V to solve for ξ

Then Substitute for ξ in the action and integrating over $y = r_c y$, we find a potential for the modulus r_c

Adjusting the parameters V_0, V_1, V_2, we find the stable minima for the radion along with the desired warp factor

1. The modulus field in the RS scenario can be stabilized in purely geometrical way
2. Appealing to plausible quantum corrections to the Einstein-Hilbert action, we trade the higher derivatives of the metric tensor for an equivalent scalar field with a complicated potential form and a nonminimal coupling to gravity
3. On going over to the Einstein frame (characterized by a nonminimal coupling), the corresponding potential is seen to have a local minimum leading to an negative effective bulk cosmological constant, and a fluctuation field with a naturally small mass
4. The resulting framework leads to the stabilization of the modulus without the need to appeal to boundary localized interactions. The correct hierarchy is obtained for a wide range of parameters
5. The mechanism offers a natural way out of the tension between the theoretical expectations for the KK-graviton masses and the strong bounds obtained at the LHC

Conclusion

1. Gravity in extra dimensions can play a significant role in BSM
2. This leads to search for extra dimensions in LHC
3. Warped geometry models with Einstein's gravity in the bulk – some tension with recent ATLAS data for kk graviton
4. Include higher curvature quantum gravity effects in the bulk which are free of ghosts
5. These effects can restore the candidature of warped geometry model for certain choices of the parameter of higher curvature terms
6. The effects of such terms should be examined in other scenario
7. Gravity in extra dimensions are projected to be strong candidates to make standard model natural
8. However the fine tuning issue pops up in different guise.
9. Explanation by 'Landscape scenario' – An 'escape' rather than a 'solution' of the problem